SISTEMAS DE ECUACIONES DIFERENCIALES DE PRIMER ORDEN CON COEF. CTES.

$$\frac{dx_1(t)}{dt} = a_{11}x_1(t) + a_{12}x_2(t) + b_1(t)$$

$$\frac{dx_2(t)}{dt} = a_{21}x_1(t) + a_{22}x_2(t) + b_2(t)$$

incógnitas $\left| \begin{array}{l} x_1(t) \\ x_2(t) \end{array} \right.$

$$x_1(0) = xo_1$$

$$x_2(0) = xo_2$$

Ejemplo.

I $\quad \dfrac{dx_1}{dt} = -7x_1 + x_2 + 5 \quad ; \quad x_1(0) = 0$

II $\quad \dfrac{dx_2}{dt} = -2x_1 - 5x_2 - 37t \quad ; \quad x_2(0) = 0$

Método de convertir el sistema en una ecuación lineal de orden "n"

Despejar x_2 en I

$$\boxed{x_2 = \frac{dx_1}{dt} + 7x_1 - 5} \quad \text{III}$$

derivar

$$\frac{dx_2}{dt} = \frac{d^2x_1}{dt^2} + 7\frac{dx_1}{dt} + (0) \quad \text{IV}$$

Sustituir III & IV en II

$$\left[\frac{d^2x_1}{dt^2} + 7\frac{dx_1}{dt}\right] = -2x_1 - 5\left[\frac{dx_1}{dt} + 7x_1 - 5\right] - 37t$$

$$\frac{d^2x_1}{dt^2} + 7\frac{dx_1}{dt} + 5\frac{dx_1}{dt} + 2x_1 + 35x_1 = 25 - 37t$$

$$\boxed{\frac{d^2x_1}{dt^2} + 12\frac{dx_1}{dt} + 37x_1 = -37t + 25}$$

$$\frac{d^2x_1}{dt^2} + 12\frac{dx_1}{dt} + 37x_1 = -37t + 25$$

E(A)C.

$$m^2 + 12m + 37 = 0$$

$$m_{1,2} = \frac{-12 \pm \sqrt{144 - 4(37)}}{2}$$

$$= \frac{-12 \pm \sqrt{144 - 148}}{2}$$

$$= \frac{-12 \pm \sqrt{-4}}{2} \Rightarrow -6 \pm i$$

$$x_{11} = e^{-6t}\cos(t)$$

$$x_{12} = e^{-6t}\operatorname{sen}(t)$$

$$x_1(t) = C_1 e^{-6t}\cos(t) + C_2 e^{-6t}\operatorname{sen}(t)$$

Homogenea

$$x_1(t) = A(t)e^{-6t}\cos(t) + B(t)e^{-6t}\operatorname{sen}(t)$$

SG

$$x_1(t) = C_1 e^{-6t}\cos(t) + C_2 e^{-6t}\operatorname{sen}(t) - t + 1$$

$$x_2(t) = (C_1 + C_2)e^{-6t}\cos(t) + (C_2 - C_1)e^{-6t}\operatorname{sen}(t) - 7t + 1$$

$$x_1(0) \Rightarrow C_1(1)(1) + C_2(1)(0) - (0) + 1 = 0$$

$$C_1 + 1 = 0 \quad \boxed{C_1 = -1}$$

$$x_2(0) \Rightarrow (C_1 + C_2)(1)(1) + (C_2 - C_1)(1)(0) - 7(0) + 1 = 0$$

$$C_1 + C_2 + 1 = 0 \quad \boxed{C_2 = 0}$$

Sol Part

$$\boxed{\begin{aligned} x_1(t) &= -e^{-6t}\cos(t) - t + 1 \\ x_2(t) &= -e^{-6t}\cos(t) + e^{-6t}\operatorname{sen}(t) - 7t + 1 \end{aligned}}$$

Sistema

$$\frac{dx_1(t)}{dt} = a_{11} x_1(t) + a_{12} x_2(t) + b_1(t)$$

$$\frac{dx_2(t)}{dt} = a_{21} x_1(t) + a_{22} x_2(t) + b_2(t)$$

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}$$

$$\boxed{\frac{d}{dt}\bar{x}(t) = A_1 \times \bar{x}(t) + \bar{b}(t)}$$